Lecture 1. Problems.

1. From

$$\hat{j}_-|j,m+1\rangle = \sqrt{(j-m)(j+m+1)}|j,m\rangle \ , \ \hat{j}_+|j,m\rangle = \sqrt{(j-m)(j+m+1)}|j,m+1\rangle$$

   construct the matrices of the operator $\hat{j}$ for $j = \frac{1}{2}$ (the Pauli matrices) and $j = 1$.

2. * Expand the functions $\cos^4\theta$ and $\cos\theta Y_{lm}(\theta,\phi)$ in terms of the spherical harmonics $Y_{lm}(\theta,\phi)$.

3. Suppose two particles with the spins $s_1 = 1$ and $s_2 = 2$ are in the state, where $m_1 = m_2 = 0$. Find the possible values of the total spin of the two particles $\hat{s} = \hat{s}_1 + \hat{s}_2$.

4. Two electrons are in the atomic f-orbital ($l_1 = l_2 = 3$). Find the possible values of the total orbital momentum $\hat{l} = \hat{l}_1 + \hat{l}_2$ compatible with the value of the total spin $s = 0$ and $s = 1$.

5. Determine the relative weight of the $m_s = 1/2$ and $m_s = -1/2$ spin projection components in the $|d_{5/2}\rangle$ single-particle wave function characterized by $m = 3/2$.

6. In nuclear physics the collective vibrations of nuclear surface can have multipolarity $\lambda = 2, 3, ...$ (quadrupole, octupole,...). Each vibrational quantum can be considered as a phonon with the momentum λ and the parity $(-1)^\lambda$. Find the possible values of the total momentum in the system of two (three) quadrupole phonons? octupole phonons?

7. Using $6j$-symbols express the wave function $|j_1 j_2(J_{12}), j_3; JM\rangle$ in terms of the wave functions corresponding to the following coupling schemes:
   (a) $\hat{j}_1 + (\hat{j}_2 + \hat{j}_3)$
   (b) $(\hat{j}_3 + \hat{j}_2) + \hat{j}_1$
   (c) $\hat{j}_3 + (\hat{j}_2 + \hat{j}_1)$

8. Transform the wave function in jj-coupling scheme $|s_{1/2}d_{3/2}; J = 2\rangle$ of two electrons to the wave function in the LS-coupling scheme.

9. Transform the wave function in LS-coupling scheme $|f^2;\ {}^1G_4\rangle$ of two electrons to the wave function in the jj-coupling scheme.

10. Determine the relative weight of the $S = 0$ and $S = 1$ intrinsic spin components in the $|d^2_{5/2}; J = 2\rangle$ two-particle wave function.