Lecture 2. Problems.

1. Find out if the set of all positive real numbers form a group and if so what the law of composition is.

2. Find out if the set of all complex numbers form a group and if so what the law of composition is.

3. Prove that in a multiplication table no elements can occur more than twice in a row or in a column.

4. What are the symmetry groups of the molecules: H_2O, NH_3, CH_4, UF_6?

5. Construct the multiplication table for the point symmetry group $\mathbf{C}_4$.

6. Construct the multiplication table for the symmetric group $\mathbf{S}_3$ (the group of permutations of three identical particles) whose elements are given by (3).

7. Show that the three elements of the symmetric group $\mathbf{S}_3$:
$$E, \quad \pi_1 \quad \text{and} \quad \pi_2$$
defined by (3) in Lecture 2, form a group which is isomorphic to the point symmetry group $\mathbf{C}_3$.

8. Show that all six elements of the symmetric group $\mathbf{S}_3$ defined in (3) form a group which is isomorphic to the point symmetry group $\mathbf{C}_{3v}$.

9. Obtain the relations between the parameters: (i) (11) from (10); (ii) (15) from (14).

10. Show that all matrices
$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{1}$$
produced by the continuous variation of θ from 0 to 2π form a continuous group (the **SO(2)** group).

11. Show that the real orthogonal $(n \times n)$ matrices whose determinants are equal to -1 do not form a group.

12. How many parameters specify the group $\mathbf{SO(n)}$?

13. Show that all matrices
$$\begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \tag{2}$$
leave invariant $x^2 - y^2$ and thus form a continuous group **SO(1,1)**.