Lecture 3. Problems.

1. Construct the representation of the group $\mathbf{D}_3$ in a 3-dimensional space formed by the vectors $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$.

2. Elements of the group $\mathbf{C}_{4v}$ can be divided into 5 classes. How many irreducible representations has this group? What are their dimensions? (See definitions of all point symmetry groups in the previous lecture).

3. Elements of the group $\mathbf{T}$ can be divided into 4 classes. How many irreducible representations has this group? What are their dimensions?

4. Elements of the group $\mathbf{O}$ can be divided into 5 classes. How many irreducible representations has this group? What are their dimensions?

5. How many irreducible representations has the group $\mathbf{C}_4$? Find their dimensions and characters, using the result of Example 1 of the section 1.4.

6. Construct the representation of the group $\mathbf{C}_{3v}$ in the 3-dimensional space formed by the functions $f_1 = x^2$, $f_2 = y^2$, $f_3 = xy$ (continue the procedure started in Example 3 of the section 1.4). Decompose this representation into irreducible components using formula (49) and the known characters of the irreducible representations.

7. Find the matrices of the direct product of two representations of the group $\mathbf{C}_{3v}$: $D^{(2)}(G)$ and $D^{(3)}(G)$.

8. Decompose the direct product of two representations $D^{(3)}(G)$ of the group $\mathbf{C}_{3v}$ into irreducible components.