Lecture 5. Problems.

1. The state with $J = 4$ decays to the state $J = 2$. Which multipolarities of the radiation are possible?

2. In the laboratory system the particle with the orbital angular momentum $l = 2$ has the value of the momentum projection on z-axis $m = 1$. Find the probability $W(m')$ that the projection of this moment on the axis which is in $\theta = 60^o$ to z-axis is equal to m' $(m' = -2, -1, 0, 1, 2)$.

3. Find a reduced matrix element of the angular momentum operator $\langle j||\hat{J}||j\rangle$ (in these notations $\hat{J}^2|jm\rangle = j(j+1)|jm\rangle$).

4. Using the property of the spherical harmonics,

$$\langle lm|Y_{kq}(\theta,\phi)|l'm'\rangle = \sqrt{\frac{(2l'+1)(2k+1)}{4\pi(2l+1)}}(l'm'kq|lm)(l'0k0|l0) \ , \tag{1}$$

   find the reduced matrix element $\langle l||Y_k||l'\rangle$.

5. Find the selection rules for the electric dipole operator in a system of cubic symmetry **O** (the transformation properties of operators x, y, z are given in the tables of characters for point symmetry groups).

6. How will the rule (49) from Lecture 5 change, if we take into account the spin-spin interaction of electrons of an atom, i.e. the terms of the form

$$H_3 = \xi(r)(\vec{S}\cdot\vec{L})^2 \ . \tag{2}$$

7. The operator equivalent to the atomic quadrupole operator (55) can be constructed from the components of the angular momentum operator:

$$\hat{Q}_{ik} = \frac{3Q}{2J(2J-1)}\left(\hat{J}_i\hat{J}_k + \hat{J}_k\hat{J}_i - \frac{2}{3}\hat{J}^2\delta_{ik}\right) \ . \tag{3}$$

   Calculate the matrix element $\langle JM|Q_{zz}|JM\rangle$.

8. Using formula (60) and the results of problem 1, calculate the matrix elements

$$\begin{array}{l}\langle j_1j_2;JM|(\vec{s}_1\cdot\vec{s}_2)|j_1'j_2';J'M'\rangle \ , \\ \langle l_1s_1;j_1m_1|(\vec{l}\cdot\vec{s})|l_2s_2;j_2m_2\rangle \ ,\end{array} \tag{4}$$

   where $\vec{j}_i = \vec{l}_i + \vec{s}_i$.