Lecture 6. Problems.

1. Obtain formula (35) from formula (33).

2. Assuming that the single-particle radial wave functions are constant in the nuclear interior and 0 outside a nucleus, i.e.
$$R(r) = \begin{cases} \sqrt{\frac{3}{R_0^3}} & \text{for } \; 0 < r < R_0 \\ 0 & \text{for } \; r \geq R_0 \; , \end{cases} \tag{1}$$
calculate the single-particle values for $B(\mathcal{E}L)$ from (35) for $j_i = L + 1/2$, $j_f = 1/2$ (Weisskopf estimates).

3. Calculate the reduced probability of the $\mathcal{E}2$-transition from the first excited $1/2^+$ state to the ground state $5/2^+$ in ^{17}O and ^{17}F within the oscillator shell model, taking into account that
$$\langle r^2 \rangle = \int R^*_{2s_{1/2}}(r) r^4 R_{1d_{5/2}}(r) dr \approx 9.5 \text{ fm}^2 \; . \tag{2}$$
Comparing the results with the experimental value $B(\mathcal{E}2) = 6.3$ e^{2}fm^4 for ^{17}O and $B(\mathcal{E}2) = 64$ e^{2}fm^4 for ^{17}F, extract the values of the neutron and proton effective charges within the sd-shell.

4. Calculate the reduced probability of the quadrupole moment Q of the ground state $3/2^-$ of ^{9}Li within the oscillator shell model without configuration mixing. Express the results in terms of the radial integrals.

5. Find the magnetic dipole moment of an odd-A nucleus in which the last nucleon occupies the state $|nlsjm; tm_t\rangle$ within the oscillator shell model without configuration mixing.

6. Calculate the magnetic dipole and the electric quadrupole moments of the ground state of the ^{31}Al within the oscillator shell model without configuration mixing.

7. From the experimental spectra of ^{41}Ca and ^{41}Sc find the single-particle energies of a neutron and a proton in the $(1f2p)$-shell with respect to the core of ^{40}Ca. From the experimental spectra of ^{39}Ca and ^{39}K find the single-hole energies of a neutron-hole and a proton-hole in the $(1d2s)$-shell with respect to the core of ^{40}Ca.

8. From the experimental spectra of ^{91}Zr and ^{91}Nb find the single-particle energies of a neutron in the $(2d_{5/2}3s_{1/2}2d_{3/2}1g_{7/2}1h_{11/2})$-shell model space and a proton in the $(1g_{9/2})$-orbital with respect to the core of ^{90}Zr.

9. From the experimental spectra of ^{209}Pb and ^{209}Bi find the single-particle energies of a neutron in the $(2g_{9/2}1i_{11/2}1j_{15/2}3d_{5/2}4s_{1/2}2g_{7/2}3d_{3/2})$-shell model space and a proton in the $(1h_{9/2}2f_{7/2}2f_{5/2}3p_{3/2}1i_{13/2}3p_{1/2})$-shell model space with respect to the core of ^{208}Pb. From the analysis of the experimental spectra of ^{207}Pb and ^{207}Tl find the single-hole energies of a neutron $(1h_{9/2}2f_{7/2}2f_{5/2}3p_{3/2}1i_{13/2}3p_{1/2})$-shell model space and a proton in the $(2d_{5/2}3s_{1/2}1h_{11/2}2d_{3/2}1g_{7/2})$-shell model space with respect to the core of ^{208}Pb.